Foot-Candles: Photometric Units

More footnotes on optical topics.
***David Gibson** describes the confusing range of photometric units.*

A discussion of photometric units may seem out of place in an electronic journal but engineers frequently have to use light sources and detectors. The units of photometry are some of the most confusing and least standardised of units.

Photometric units are not difficult to understand, but can be a minefield to the uninitiated since many non-SI units are still in use, and there are subtle differences between quantities with similar names, such as *illumination* and *luminance*.

Here I will give explanations of the more common units, but I will not digress into some of the more specialised applications. These include how photographic film speeds are defined, and why the "f/ number" of a lens is important (when it is not important, when "exposure factor" is important too). This information is necessary for the design of photometers and exposure meters, but is outside the scope of this article.

Definitions

Radiant and Luminous Flux.

Light is a form of electromagnetic energy, and its power flux can be measured in watts. This is the SI unit of power, and of ***radiant flux***. Since many measurements of light are connected with the response of the human eye, measuring power in this way does not give an indication of the 'brightness' of an object. A weighted power measurement is used, called the *lumen*. This is the SI unit of ***luminous flux***.

The adjective 'luminous' refers to the fact that the power is weighted according to the response of the human eye. Two response curves (for photopic and scotopic vision) are usually used and have been defined and standardised by the C.I.E. (Commission Internationale de l'Eclairage).

One way of defining the lumen (though not the SI method) would be to relate it to these response curves, where 1W of radiant power at 555nm (peak sensitivity) corresponds to 673 lm. (This is the new definition; prior to 1970 the figure was different).

Luminous efficiency is the ratio of luminous to radiant flux at a particular wavelength. ***Relative luminous efficiency*** is the ratio of luminous efficiency to luminous efficiency at the wavelength where the eye is most sensitive. Unfortunately, however, this term can be confused with the term ***efficacy***, which is used to describe the efficiency at converting electrical to luminous power.

Illumination, Luminous Emittance.

The ***illumination*** of a surface is the incident power flux density measured in lumens per square metre. A formal definition would be along the lines of: if a flux ΔF is incident over an area ΔS at an angle θ to the normal, then the flux density, E, is given by

$$E = \lim_{\Delta S \to 0} \frac{\Delta F}{\Delta S} \cos\theta \qquad (1)$$

Emitted flux (or ***luminous emittance***) can be defined in the same way. The lm/m^2 is given the name *lux* (abbreviation lx) when it refers to illumination, but not when it refers to luminous emittance, for which the unit lm/m^2 is retained.

Luminous Intensity ("Candlepower")

When considering the brightness of a source we need to know the angular distribution of the flux. This is called the ***luminous intensity*** and is measured in lumens per steradian[1] for which we use the name *candela*. If a flux ΔF is emitted in a solid angle $\Delta\Omega$ then the angular distribution of the flux can be given by

$$I(\theta) = \lim_{\to 0} \frac{\Delta F}{} \qquad (2)$$

Note that the lumen and candela describe the properties of a point in space. Other units are used to describe the properties of extended surfaces. Luminous intensity should always have the direction specified since it is defined as the element of flux ΔF in a cone of width $\Delta\Omega$ in direction θ.

The relationship between solid angle Ω and cone full-angle θ is given below. The approximation (θ in degrees) is true to 2% for θ<60°; 5% for θ<90°.

$$\Omega = 2\pi\left(1 - \cos \tfrac{1}{2}\theta\right) \approx 0.00024\,\theta^2 \qquad (3)$$

[1] The SI unit of solid angle is the steradian [sr] and is defined in a similar way to the radian – the area of a sphere subtended by a cone of angle 1sr is equal to the radius squared. In terms of "full angle" the cone is ≈ 65.54°.

The non-SI unit ***mean spherical candle-power*** is the intensity of a source if its light output were spread evenly in all directions. It is therefore equivalent to the flux [lm] ÷ 4π.

Luminance

The candela measures the intensity of a point source. We also need to define the properties of an extended source. Each small element ΔS of a diffuse reflective surface will scatter the incident flux ΔF and behave as if it were an infinitesimal point source. The 'brightness' of an extended surface is therefore measured in candelas per square metre.

Luminance, as the property is called, can refer to reflective surfaces or self-luminous surfaces such as a c.r.t., fluorescent tube, diffuse lamp housing, or the sky – see box. In general, luminance depends on direction. The fact that the direction can only be defined for a very small area can be a source of confusion, given that the property itself refers to large areas.

Suppose an element of area ΔS shines with an intensity ΔI_θ at an angle θ. The element of area projected at angle θ is $\Delta S \cos\theta$ so we can define the luminance in direction θ as

$$L(\theta) = \lim_{\Delta S \to 0} \frac{\Delta I_\theta}{\Delta S \cos\theta} \qquad (4)$$

There are a particular class of surfaces called ***perfectly diffuse*** or Lambert surfaces for which the intensity varies with angle in such a way that

$$\Delta I_\theta = \Delta I_n \cos\theta \qquad (5)$$

where I_n is the luminous intensity in the normal direction. For these surfaces it can be seen that the luminance is constant with direction, being equal to $\Delta I_n / \Delta S$. Such a surface appears equally bright from all directions.

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The brightness of the sky

On a clear day, illumination from the sun is about 50,000lx, but it can vary from 130,000 to 10,000lx. The moon has an illuminance of 0.1 lx. Indoor lighting is normally a few hundred lux. Considering not just the sun, but the luminance of the sky as a whole ; a clear noon sky, near the horizon, has a luminance of around 10,000cd/m^2. A cloudy sky at sunset is around 10cd/m^2.

Fluorescent Tube Drivers

*A selection of references to circuits for driving fluorescent tubes collected by **David Gibson**.*

Traditionally, most battery-powered fluorescent lights used a one-transistor oscillator driving a transformer. It was not only inefficient, but the output waveform had a d.c. bias that caused one end of the tube to darken. But technology has moved on since those days. Electronic ballast is now big business – the transistors are fast and efficient and the coupling Cs (to eliminate the d.c. bias) are of a high quality. Zetek, a semiconductor firm, now manufacture a range of transistors specifically designed for high frequency lamp ballasts. This is one of many cases where it could pay to re-visit an "old" problem in the light of "new" technology. A quick search of my index to *Electronics World* reveals the following recent articles. For *EW* back-issues telephone 0181-652 3614

DC-to-DC Converter Transistors,
Zetex AN81 application note,
June 1992, p517 & Feb. 1993, p165 [reprint]

The application note AN81 covers the use of the company's range of E-line bipolar and mosfet transistors in DC-to-DC converters providing up to 10W for small equipment such as fluorescent tubes and flashguns.

Fluorescent Lamp Ballast,
IR application note, October 1994, p837

The IR2151 is a high voltage, high speed, self-oscillating driver for power mosfets and IGBTs. [also brief mention, April 1995, p349]

Fluorescent Backlighting,
LT Application, Dec. 1993, p1037

The circuit using an LT1172 drives two cold cathode fluorescent back-lighting lamps at 92% efficiency. It also features dimming and shut-down facilities to help maximise battery life.

IGBT Switching Reduces Ballast Size,
Oct. 1994, p869

Zetek IGBT types ZCN0545 and ZCP0545 are used in an 11W off-line fluorescent lamp ballast. They are so efficient they can replace TO220/126 bipolar or mosfet transistors and will provide savings in both cost and circuit volume.

Lighting Switches, by Martin Eccles Sept 1994, p752 [reviewed in CREGJ **21**]

A look at some efficient switching designs for applications ranging from emergency beacons to LCD back-lighting. Modern high-performance chip designs make it possible to drive a fluorescent lamp using a transistor with tiny footprint.

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Relationship between Luminance and Luminous Emittance

Both these quantities describe the radiated energy from a surface, and it is to be expected that there is a relationship between them. For a Lambert surface it can be shown that the relationship is

$$\text{Luminous emittance } [\text{lm/m}^2] = \pi \times \text{Luminance } [\text{cd/m}^2]$$

If the surface is perfectly reflective, as well as being perfectly diffuse then the emitted flux must equal the incident flux, and this gives the relationship between illumination and luminance:

$$\text{Luminance } [\text{cd/m}^2] = 1/\pi \times \text{Illumination } [\text{lm/m}^2] \times \text{Reflectivity } [\%]$$

Luminance can be a difficult quantity to get to grips with. For example, it is the luminance of a surface that is significant in discussions of cameras, lenses and the human eye. A scene luminance in cd/m^2 gives rise, on the other side of the lens, to an illumination in lux. This is why, for example, that the photopic and scotopic curves are defined for a particular scene luminance and not in terms of illumination or luminous emittance.

Relationship between intensity and illumination

The area subtended by a cone of solid angle Ω is $r^2\Omega$ so a source of intensity I cd causes an illumination of I/r^2 lux at a distance r.

Example: An LED has an on-axis intensity of 3cd and it is shone onto a surface 2m away. The illumination at the centre of the beam will therefore be 0.75 lx. The surface is perfectly diffuse, and has a reflectance of 10% so the resulting luminance will be 24mcd/m^2.

Units of Measurement

Historically the candle (forerunner to the candela) was defined, not surprisingly, as the brightness of a candle. Equally unsurprising is the fact that this is a difficult standard to maintain. The SI definition of the candela is repeatable, but difficult to set up in practice. It is derived from the luminance of a black body surface at the temperature of freezing platinum (now accepted to be 2045.5K), at standard pressure; the luminance of which is defined to be 60cd/m^2 in the normal direction. The lumen, which is in some respects a more 'basic' unit, is derived in terms of the candela.

Some of the SI units described above have been given non-standard names, such as the *nit* for cd/m^2. Other names exist for sub-multiples of basic units; for example the *phot*, which is 100µlx (i.e. 1 lm/cm^2) and the *stilb*, which is 100µcd/m^2 (i.e. 1cd/cm^2). These units are non-preferred. There are also the 'candle' units which, confusingly, refer to illumination, and not to luminance A *foot-candle* is a lumen per square foot, whilst a *metre-candle* is simply a lumen per square metre, or lux.

The relationship between luminous intensity and luminance mentioned above has lead another series of units. A *Lambert* is 1/π cd/cm^2, thus a surface with an illumination of 1lm/cm^2 (1 phot if you must) has a luminance of one Lambert. This unit is large and so the milli-Lambert (mL) at 3.183cd/m^2 is more commonly used.

There is also the *foot-Lambert*, which is 1/π cd/ft^2, and the *metre-Lambert* or *apostilb*, which is 1/π cd/m^2. The *apostilb* is thus the luminance of a perfectly diffuse and perfectly radiating surface with an illumination of 1 lx. The *skot* is a milli-apostilb. This rapidly gets very confusing!

There are also the *Troland*, and *Luxon*, which are the illumination on the retina from a scene of luminance 1cd/m^2 when viewed through a pupil of aperture 1mm.

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